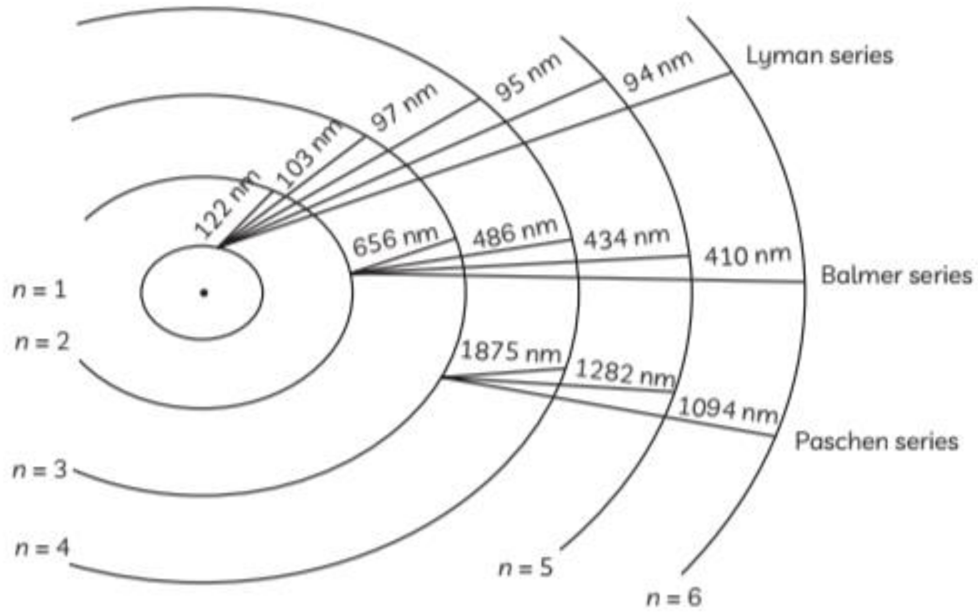
Structure of Atom

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. A hydrogen atom consists of an electron orbiting its nucleus. The electromagnetic force between the electron and the nuclear proton leads to a set of quantum states for the electron, each with its own energy. These states were visualised by the Bohr model of the hydrogen atom as being distinct orbits around the nucleus. Each energy state, or orbit, is designated by an integer, n as shown in the figure. The Bohr's model was later replaced by quantum mechanics in which the electron occupies an atomic orbital rather than an orbit, but the allowed energy levels of the hydrogen atom remained the same as in the earlier theory.



(A) Which series of lines of the hydrogen spectrum lies in the visible region?
(B) Wavelengths of different radiations are given below:

(I) λ= 300 nm
(II) λ = 300 μm
(III) λ = 3 nm
(IV)λ =30 Å

Arrange these radiations in the increasing order of their energies.

(C) What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition n = 4 to n = 2 of He+ spectrum?

Ans. (A) Balmer series
(B) (IV) > (11) > (1)> (11)
The lower the wavelength, the higher the energy.
(C) For the Balmer transition, n = 4 to n = 2 in a He+ ion, we can write.

$$\frac{1}{\lambda} = Z^2 R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= Z^2 R_H \left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= \frac{3}{4} R_H \qquad ...(i)$$

For a hydrogen atom,

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad ...(ii)$$

Equating equations (i) and (ii), we get

$$\frac{1}{n_1^2} = \frac{1}{n_2^2} = \frac{3}{4}$$

This equation gives $n_1 = 1$ and $n_2 = 2$. Thus,
the transition n = 2 to n = 1 in hydrogen atom will have the same wavelength as the transition, n = 4 to n = 2 in He+.

2. Based on the wave model of light, physicists predicted that increasing light amplitude would increase the kinetic energy of emitted photoelectrons, while increasing the frequency would increase measured current. Contrary to the predictions, experiments showed that increasing the light frequency increased the kinetic energy of the photoelectrons, and increasing the light amplitude increased the current. Based on these findings, Einstein proposed that light behaved like a stream of particles called photons with an energy of E = hv
The work function, D, is the minimum amount of energy required to induce photoemission of electrons from a metal surface, and the value of depends on the metal. The energy of the incident photon must be equal to the sum of the metal's work function

and the photoelectron kinetic energy:

$E_{photon} = K.\ E_{electron} + \Phi$

(A) Radiation of 2500 Å falls on a metal with a work function of 4 eV. The kinetic energy of the fastest photoelectron will be:

(a) 3.22×10^{-19}J

(b) 1.55×10^{-19}J

(c) 4×10^{-19} J

(d) 2.5×10^{-19}J

(B) When a photoelectric experiment is conducted, the number of electrons released is proportional to the:

(a) Intensity of light

(b) Brightness of light

(c) Both (a) and (b)

(d) None of the above

(C) For an ejected electron, kinetic energy is:

(a) same as the frequency of the radiation from electromagnetic fields.

(b) proportional to the frequency of the radiation from electromagnetic fields.

(c) greater than the frequency of the radiation from electromagnetic fields.

(d) inversely proportional to the frequency of the radiation from electromagnetic fields.

(D) In an orbit, magnitude of kinetic energy is equal to:

(a) half of the potential energy

(b) twice of the potential energy

(c) one-fourth of the potential energy

(d) none of the above

(E) The minimum energy required to remove an electron is called:

(a) Stopping potential

(b) Kinetic energy

(c) Work function

(d) None of these

Ans. (A) (b) 1.55×10^{-19}J

Explanation:

Given, $E_0 = 4 \text{ eV} = 4 \times 1.60 \times 10^{-19} \text{J}$

We know that, $c = 3 \times 10^8 \text{ m/s} [1Å = 10^{-10} \text{m}]$

$$\because \quad E = h\nu = \frac{hc}{\lambda}$$

$$\therefore \quad E = \frac{6.63 \times 10^{-34} \text{ Js} \times 3 \times 10^8 \text{ ms}^{-1}}{2500 \times 10^{-10} \text{ m}}$$

$$= 7.95 \times 10^{-19} \text{ J}$$

$\therefore$ Kinetic energy of electron emitted

$$= (7.95 - 6.4) \times 10^{-19} \text{ J}$$

$$= 1.55 \times 10^{-19} \text{ J}$$

(B) (c) Both (a) and (b)

Explanation: Intensity and brightness of light decide the number of ejected electrons.

(C) (b) proportional to the frequency of the radiation from electromagnetic fields.

Explanation: Kinetic energy of ejected electron is proportional to frequency of the radiation from electromagnetic fields.

(D) (a) half of the potential energy

Explanation: We know that the Kinetic

energy in an orbit $= \dfrac{Ze^2}{8\pi\varepsilon_0 r}$

Also, the Potential energy in an orbit

$$= -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

From these two we have, $\text{K.E.} = \frac{1}{2}\text{P.E.}$

So, the magnitude of kinetic energy in an orbit is equal to half of the potential energy.

3. In 1924, de Broglie suggested that if the light is known to consist of waves and under certain situations assume the aspect of a particle then the particle should also behave like a wave. He based his reasoning on the assumption that nature possesses symmetry and that the two physical entities matter and waves must be symmetrical also. de Broglie took the quantum idea of emission of energy of a photon of radiation of a certain frequency which can be obtained using the equation given by him. That equation is called de Broglie's equation and this wavelength is called de Broglie's wavelength. The novel idea of this equation is the wave- particle nature of matter with the relative motion of particles and certain wave links with it. This idea leads to the dual nature of light also.

(A) de Broglie equation is obtained by a combination of:

(a) Interference

(b) Diffraction

(c) Einstein's theory of mass-energy equivalence

(d) Photoelectric effect

(B) Wave nature of the electron is shown by:

(a) Photoelectric effect

(b) Crompton effect

(c) Diffraction experiment

(d) None of the above

(C) de Broglie wavelength of a particle is:

(a) Proportional to mass

(b) Inversely proportional to momentum

(c) Inversely proportional to plank constant

(d) Proportional to velocity

(D) A 0.66 kg ball is moving with a speed of 100 m/s. the associated wavelength will be (h= 6.6 x 10-^{34}Js):

(a) 6.6 x 10^{-34}

(b) 6.6 x 10^{-36}

(c) 1.6 x 10^{-34}

(d) 1 x 10^{-35}

(E) The position of both the electron and the Helium atom is known within 1 nm and the momentum of the electron is known within 50 x 10-26 kg ms-1. The minimum uncertainty in the measurement of the momentum of the helium atom is:

(a) 50 x 10^{-26} kg ms^{-1}

(b) 50 kg ms-^{1}

(c) 80 kg ms-1

(d) 60 x 10-26 kg ms-1

Ans. (A) (c) Einstein's theory of mass-energy equivalence

Explanation: Einstein's theory of mass equation and Planck's constant contributes to the de Broglie equation.

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = mc^2$$

As the smaller particle exhibits dual nature, and energy being the same, de Broglie equated both these relations for the particle moving with velocity 'v. From these two equations:

$$E = \frac{hc}{\lambda} = mv^2$$

then, $\lambda = \frac{h}{mv}$

(B) (c) Diffraction experiment

Explanation: According to de Broglie, the wave nature is shown by diffraction experiments. Louis de Broglie in his thesis suggested that any moving particle, whether microscopic or macroscopic will be associated with a wave character. It was called 'Matter Waves'. He further proposed a relation between the velocity and momentum of a particle with the wavelength, if the particle had to behave as a wave.

(C) (b) Inversely proportional to momentum

Explanation: According to the de Broglie equation, the wavelength is inversely proportional to the mass and the velocity.

de broglie equation: $\lambda = \frac{h}{mv}$

(D) (d) $1x10^{-35}$

Explanation: According to the de Broglie

equation $\lambda = \frac{h}{mv}$

$$\lambda = \frac{6.6 \times 10^{-34}}{0.66 \times 100}$$

$$= 1 \times 10^{-35} \text{ m}$$

(E) (a) 50×10^{-26} kg ms-1

Explanation: The product of uncertainties in the momentum and position of a subatomic particle $= \frac{h}{4\pi}$. Since position (Δx) is the same for both electron and the Helium atom so, Ap must be the same for both the particles ie., 50×10^{-26} kg ms^{-1}.

4. A total of four quantum numbers are used to describe completely the movement and trajectories of each electron within an atom. The combination of all quantum numbers of all electrons in an atom is described by a wave function that complies with the Schrödinger equation. Each electron in an atom has a unique set of quantum numbers; according to the Pauli Exclusion Principle, no two electrons can share the same combination of four quantum numbers. Quantum numbers are important because they can be used to determine the electron configuration of an atom and the probable location of the atom's electrons. Quantum numbers are also used to understand other characteristics of atoms, such as ionisation energy and the atomic radius. In atoms, there are a total of four quantum numbers: the principal quantum number (n), the orbital angular momentum quantum number (1), the magnetic quantum number (m), and the electron spin quantum number (m.). The principal quantum number, n, describes the energy of an electron and the most probable distance of the electron from the nucleus. In other words, it refers to the size of the orbital and the energy level at which an electron is placed. The number of subshells, or l, describes the shape of the orbital. It can also be used to determine the number of angular nodes. The magnetic quantum number, my describes the energy levels in a subshell, and m, refers to the spin on the electron, which can either be up or down.

(A) Answer the following questions:

(i) What do the quantum numbers $+\frac{1}{2}$ and $-\frac{1}{2}$ for electron spin represent?

(ii) Which quantum number represents the size and shape of the subshell?

(B) In which order, the energy of different subshells can be arranged for the given value of n?

(C) In dxy subshell, at what angle there is the probability of finding an electron?

(i) The quantum numbers $+\frac{1}{2}$ and $-\frac{1}{2}$

electron spin represents the clockwise and anti-clockwise spin with no significant similarities.

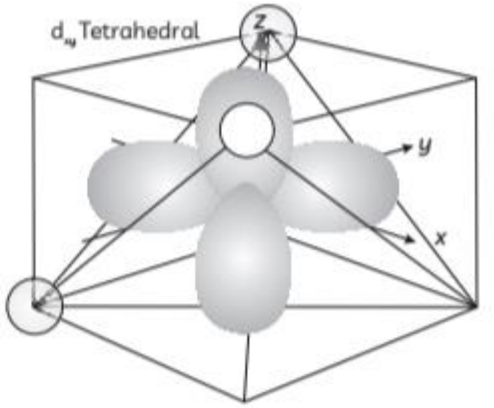
(ii) The primary quantum number 'n' represents the size of an orbital. The comparative distance from the nucleus as well as the energy levels is shown by the primary quantum number.

The azimuthal quantum number 'l' represents the shape of an orbital and also determines its angular momentum.

(B) For the given value of n, the energy of different subshells can be arranged according to Aufbau's principle. As per the Aufbau principle, the filling of electrons is in the order 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 4f, 5d, 6p, 7s...

So, the right filling sequence of the subshell is f>d>p>s.

(C) According to the d-orbital subshell representation, the probability of finding the electrons in dxy orbital is along the X-axis at an angle of 45°.



d$_{xy}$ tetrahedral orbital